LXXXI. *Some Considerations on a late Treatise intituled,* A new Set of Logarithmic Solar Tables, *&c. intended for a more commodious Method of finding the Latitude at Sea, by Two Observations of the Sun; by* H. Pemberton, *M. D. R. S.* Lond. *et R. A.* Berol. *S.*

Read Nov. 20, 1760.

AS it happens not unfrequently at sea for the unseasonable intervention of clouds to prevent the ordinary method of determining the ship's latitude by the sun's meridian altitude, even when it is of primary consequence, that the true latitude should be known; a problem for remedying this disappointment is stated in many treatises of navigation, for finding the latitude of a place by any two altitudes of the sun, with the interval of time between them.

A problem similar to this is proposed, and solved instrumentally upon a globe, by a very early writer, Petrus Nonius (*a*), namely, to find the latitude by two altitudes of the sun, and the angle made by the azimuth circles passing through the sun, when the altitudes are taken. And since more commodious and accurate instruments for measuring time have been invented, than were known to this author, the other problem has been proposed for the same pur-

(*a*) In Libr. De Observ. Regul. et Instrum. Geometr. lib. ii. c. 14.

pose,

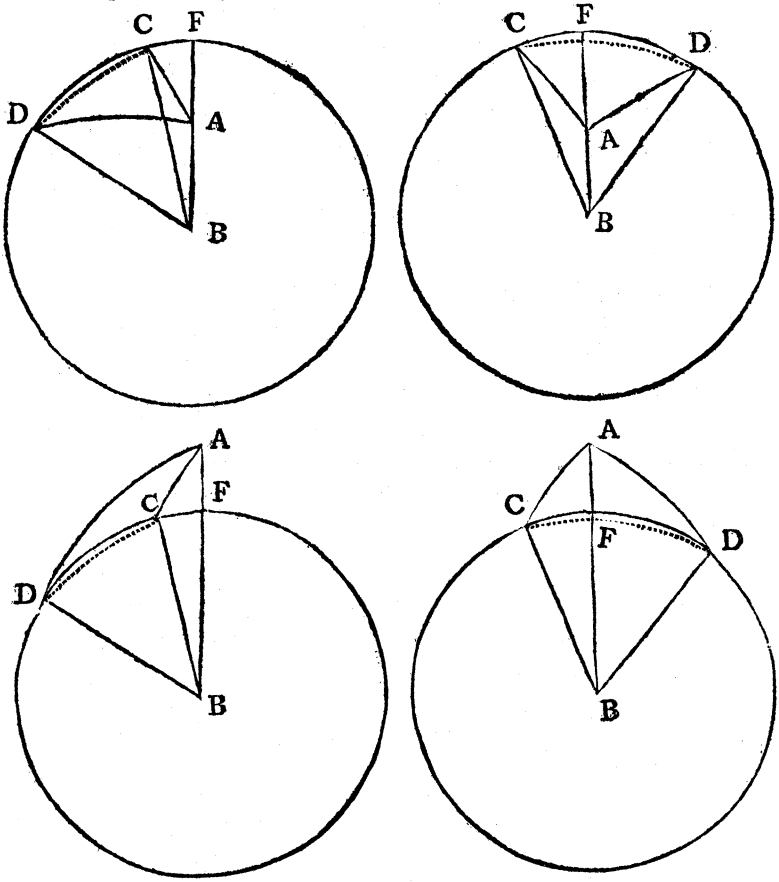
pofe, of which a conftruction upon the principles of the ftereographic projection of the fphere is exhibited by Mr. Collins, in his Mariner's Plain Scale new planed (*b*). And as the direct method of folving both thefe problems by numbers requires a diverfity of trigonometrical operations, a fet of tables has lately been publifhed for a more compendious way of computation in the problem, where the interval of time is given, whereby the fhip's true latitude may be very expeditioufly derived from the fhip's dead reckoning, provided the obfervations are made within certain limits of time.

But however worthy of notice this method may be, new tables for the purpofe are altogether unneceffary. It confifts of two parts: the firft computes, from the latitude exhibited by the dead reckoning of the fhip, the diftance from noon of the middle time between the obfervations, and thence the time of either: the fecond operation computes, from one of thefe obfervations, what fhould be the fun's meridian altitude, had the fhip's reckoning given the true latitude; but if the latitude affumed from that reckoning is erroneous, the altitude thus computed will not be conformable to it; however, if the times for the obfervations are properly chofen, it will much better agree to the true latitude, and thereby the affumed latitude may be more or lefs corrected.

But both thefe operations are an immediate confequence from the propofition in fpherical trigonometry, ufually delivered under the name of the fourth axiom, which is this; That the fquare of the radius is to the rectangle under the fines of the fides con-

(*b*) Book iii. p. 35.

taining any angle, as the verfed fine of that angle to the difference between the verfed fines of the third fide, and of the difference between the fides contain-taining the angle.



Hence if A be the zenith of any place; B the elevated pole; C, D two places of the fun in the parallel of declination D F; by this propofition in the triangle A B C,

Rad.

$$\text{Rad.}^{q} : \text{ſin. AB} \times \text{ſin. BC (coſ. lat.} \times \text{coſ. decl.} \odot) :: \text{verſ. ſ. ABC : verſ. ſ. AC — verſ. ſ. AF;}$$

And, in like manner, in the triangle ABD,

$$\text{Rad.}^{q} : \text{coſ. lat.} \times \text{coſ. decl.} \odot :: \text{verſ. ſ. ABD : verſ. ſ. AD — verſ. ſ. AF;}$$

Conſequently,

$$\text{Rad.}^{q} : \text{coſ.lat.} \times \text{coſ.decl.} \odot :: \text{verſ.ſ.ABD} - \text{verſ.ſ.ABC : verſ. ſ. AD} - \text{verſ. ſ. AC.}$$

But $\frac{1}{2}$ rad. $\times \overline{\text{verſ. ſ. ABD} - \text{verſ. ſ. ABC}}$ is $=$ $\text{ſin.} \frac{ABD + ABC}{2} \times \text{ſin.} \frac{ABD - ABC}{2}$;

Therefore,

$$\text{Rad.}^{q} : \text{coſ. lat.} \times \text{coſ. decl.} \odot :: \text{ſin.} \frac{ABD + ABC}{2} \times \text{ſin.} \frac{ABD - ABC}{2} : \tfrac{1}{2}\,\text{rad.} \times \overline{\text{verſ.ſ. AD} - \text{verſ.ſ. AC}}.$$

And this is the firſt operation in the treatiſe, theſe remarks concern. For in the rectangle, the ſin. $\frac{ABD + ABC}{2}$ $\times$ the ſin. $\frac{ABD - ABC}{2}$, the firſt ſide, when the obſervations are both made on the ſame ſide of noon, and the ſecond ſide of this rectangle, when one obſervation is before noon, and the other after, is the ſine of the diſtance of the middle time between the obſervations from noon, and the other ſide is the ſine of half the diſtance between the obſervations. And in the new tables one column exhibits the arithmetical complement of the logarithmic ſine of half the elapſed time, and the next, the logarithm of twice the ſine of the middle time.

But moreover, $\frac{1}{2}$ rad. $\times \overline{\text{verſ. ſ. AD} - \text{verſ. ſ. AC}}$ being equal to ſin. $\frac{AD + AC}{2} \times$ ſin. $\frac{AD - AC}{2}$, the operation may be expreſſed more commodiouſly thus,

Rad.

Rad.^{q} : cos. lat. × cos. decl. ⊙ : : sin. $\frac{ABD + ABC}{2}$ × sin. $\frac{ABD - ABC}{2}$: sin. $\frac{AD + AC}{2}$ × sin. $\frac{AD - AC}{2}$; now requiring the common logarithmic sines only.

Again, the square of the radius being, as above, to cos. lat. × cos. decl. ⊙, as the vers. s. A B C to vers. s. A C — vers. s. A F, or as vers. s. A B D to vers. s. A D — vers. s. A F, these analogies express the second operation, which, in this treatise, is unnecessarily confined to the lesser angle A B C. The column in the table intituled Rising consists of logarithmic versed sines, which may be applied to either of the angles A B C or A B D promiscuously; for here A F being equal to the excess of B C or B D above the arch B A, assumed for the complement of the latitude, the arches A C and A D, in the preceding analogies, will be the complements of the altitudes observed, if the latitude were truly assumed, otherwise not; but the difference of their versed sines, will however be equal to the difference of the versed sines of the complements of the true latitudes; for this is supposed in the first operation. Therefore, if one of the fourth terms of these analogies be deducted from the versed sine of the complement of the greater altitude, or the other from the versed sine of the complement of the lesser, the remainder will be the same, and exhibit a versed sine for the complement of the sun's altitude different from A F, when B A is assumed different from the true latitude, and nearer to the truth, if the times for the observations are properly chosen.

But farther, the two preceding analogies may be reduced to these;

Rad.

$\text{Rad.}^q : \underline{\text{cos. lat.} \times \text{cos. decl.} \odot} :: \overline{\text{sin.} \frac{1}{2} ABC}|^q$
$: \frac{1}{2} \text{rad.} \times \overline{\text{vers. s. } AC - \text{vers. s. } AF}$,

And

$\text{Rad.}^q : \underline{\text{cos. lat.} \times \text{cos. decl.} \odot} :: \overline{\text{sin.} \frac{1}{2} ABD}|^q$
$: \frac{1}{2} \text{rad.} \times \overline{\text{vers. s. } AC - \text{vers. s. } AF}$;

Also,

Vers. s. AC − vers. s. AF = cos. AF − cos. AC, and vers. s. AD — vers. s. AF = cos. AF — cos. AD.

And thus the common tables of sines will supply the use of this column of the new tables, as well as the preceding.

The first example of this treatise, wrought by the common tables.

Here the declination of the sun is stated at 11° 17′ N. the altitude first taken, being the lesser, 46° 55′, the greater altitude 54° 7′, and the difference of time between the observations $1^h\ 25^m$, or 85^m; which, divided by 4, gives 21° 15′ for the arch of the equinoctial corresponding.

Here the observations are both before noon; therefore, in fig. 1. the latitude being assumed 46° 50′,

For the first operation.

	°	′	″	
AC =	35	53	0	
AD =	43	5	0	
Their sum =	78	58	0	
Their difference =	7	12	0	
Sine of ½ sum	(39	29	0)	9.80336
Sine of ½ difference	(3	36	0)	8.79789
Arith. com. of the sine of ½ CBD	(= 10	37	30)	0.73429
Arith. com. of the sine of BF, or the cos. of the decl. of the sun, - - - - - - - - - - - - -				0.00848
Sum				19.34402
Cos. of the lat. assumed				9.83513
Diff. = sin. $\frac{ABD + ABC}{2}$				9.50889

For the second operation.

$$\frac{ABD + ABC}{2} = 18^\circ\ 49'\ 50''$$

$$\frac{ABD - ABC}{2} = 10\ \ 37\ \ 30$$

$$ABC = 8\ \ 12\ \ 20$$

Sine of $\frac{1}{2}$ ABC (4 6 10)	8.85458 8.85458
Cos. of decl. ☉	9.99152
Cos. of the lat. assumed	9 83513
Sum − 30.00000 = log. of the $\frac{\text{vers.s. AC} - \text{vers.s. AF}}{2}$	7.53581
Vers. s. AC — vers. s. AF	0.006868
Natural cos. of AC, the complement of the greater altitude, - - - - - - - - - - - - - -	0.810212
Natural cos. of $35^\circ\ 12'\ 23''$	0.817080

Or, $\frac{ABD + ABC}{2} + \frac{ABD - ABC}{2}$ being = ABD,

Sine of $\frac{1}{2}$ ABD ($14^\circ\ 43'\ 40''$)	9.40522 9.40522
Cos. of the decl. of the sun	9 99152
Cos. of the latitude assumed	9.83513
Sum − 30.00000 = log. of the $\frac{\text{vers s. AD} - \text{vers s. AF}}{2}$	8.63709
Vers. s. AD — vers. s. AF	0 086720
Natural cos. of AD, the complement of the lesser altitude, - - - - - - - - - - - - - -	0.730361
Natural cos. of $35^\circ\ 12'\ 23''$, as before	0.817081

Thus the meridian distance of the sun from the zenith is computed at $35^\circ\ 12'\ 23''$, and this added to the sun's declination $11^\circ\ 17'$ N. gives $46^\circ\ 29'\ 23''$ for the latitude, different from that assumed, and nearer to the true latitude; which, by a second operation

operation with this latitude now found, will approach ſtill nearer to the truth, thus:

The ſum of the four firſt logarithms in the firſt of the two former operations, - - - - - - -		19.34402
Coſine of	46° 29′ 23″	9.83789
Sine of $\frac{ABD + ABC}{2}$	= 18 42 25	9.50613
$\frac{ABD - ABC}{2}$	= 10 37 30	
ABC	= 8 4 55	
$\frac{1}{2}$ ABC	= 4 2 $27\frac{1}{2}$	8.84800
		8.84800
		9.99152
		9.83789
Log. $\frac{\text{verſ. ſ. AC} - \text{verſ. ſ. AF}}{2}$		7.52541
Verſ. ſ. A C — verſ. ſ. A F		0.006704
Natural coſ. of A C, as before,		0.810212
Natural coſ. of 35° 13′ 21″		0.816916

The declination of the ſun added to this, amounts to 46° 30′ 31″, the true latitude, as directly computed, being 46° 30′ 19″.

However, it muſt not be expected, that this method of computation will always converge thus expeditiouſly to the true latitude. Had theſe obſervations been made about 3 hours ſooner in the day, and with the ſame interval between them, if the firſt altitude of the ſun had been found 18° 55′, and the ſecond 33° 11′, the latitude computed would come out wider from the truth, than that aſſumed, one exceeding, and the other falling ſhort of the true latitude, which will lie between them, and is nearly the ſame, as above. But if the leſſer altitude had

 been

been 29° 13′, and the greater 40°; in 9 degrees of north latitude the interval between the obſervations would have been nearly the ſame, as before. But if this latitude be ſought by any latitude aſſumed near it, the latitude computed according to the method above, will fall more remote from the truth, than that aſſumed, and err the ſame way.

In general, the error in the latitude aſſumed will bear to the error in the latitude computed, nearly the ratio compounded of that of the rectangle under the radius and the coſine of the diſtance from noon of the middle point of time between the two obſervations to twice ſin. $\frac{1}{2}$ ABC × ſin. $\frac{1}{2}$ ABD, and the ratio of rad. × ſin. AF to ſin. BC × coſ. AB; inſomuch that the diſtance of the middle point of time between the two obſervations from noon is to be conſidered, as the limit, where theſe computations ſhall ceaſe to converge, when the rectangle under the radius, and the coſine of this diſtance from noon ſhall be to twice ſin. $\frac{1}{2}$ ABC × ſin. $\frac{1}{2}$ ABD, (or the coſine of this diſtance to the difference between the verſed ſine of this diſtance from the verſed ſine of half CBD) as ſin. BF × coſ. AB to rad. × ſin. AF. And the errors in the aſſumed and computed latitudes fall on different ſides of the true latitude, when both the obſervations are on the ſame ſide of noon, and the zenith lies between the meridian ſun and the elevated pole; or when the obſervations are one before, and the other after noon, if the meridian ſun paſſes between the zenith and the elevated pole: otherwiſe, they fall on the ſame ſide of the true latitude.

However,

However, the direct method of computation, by the aſſiſtance of the natural ſines, will not be ſo much more operoſe than this compendium, as may at firſt ſight be imagined. For the arch of a great circle being drawn through C and D, forming the triangle BCD, if the logarithmic coſine of BC or BD be added to the logarithmic tangent of half the angle CBD, and the logarithmic ſine of BC or BD be added to the logarithmic ſine of half this angle, the firſt ſum is the logarithmic cotangent of the angle BCD, and the ſecond the logarithmic ſine of half CD, the baſe of the triangle. Then, in the triangle ACD, from the ſides, now all given, is to be computed the angle ACD, the difference between which, and the angle above found BCD, is the angle BCA, when the zenith lies between the pole, and the great circle through C, D; but when the zenith lies beyond that circle, the angle BCA is either the ſum of thoſe angles, or the ſupplement of that ſum to a circle. And, in the laſt place, if twice the logarithmic ſine of half this angle, and the logarithmic ſines of BC and AC are added together, the ſum, after thrice the logarithm of the radius has been deducted, is the logarithm of half the exceſs of the natural coſine of BC ∽ AC above the natural coſine of AB, or the natural ſine of the latitude, according to the trigonometrical axiom, which has been above referred to; for rad. $\times$ $\frac{1}{2}$ verſ. ſ. BCA is equal to $\overline{\text{ſin. } \frac{1}{2} \text{ BCA}}|^{q}$ (*c*).

(*c*) Vide Neper. Mirif. Canon. Conſtruct. Edinburg. 1619, p. 59.

The

The operation for the preceding example will be this:

Tang. ½ C B D —	9.273225	Sine ½ C B D —	9.265714
Cof. B C —	9.291504	Sine B C —	9.991524
Cotang. B C D —	9.564729	Sine ½ C D —	9.257238
	B C D = 87° 53′ 52″		½ C D = 10° 25′ 2½″

	°	′	″		
A D =	43	5	0		
A C =	35	53	0	— 0.232001	ar.co. of the fines.
C D =	20	50	5	— 0.448950	
Sum =	99	48	5		
½ Sum =	49	54	2½	— 9.883621	fines.
½ Sum — A D =	6	49	2½	— 9.074466	
			Sum	— 19.639038	
			½ Sum	— 9.819519	

This half fum is the logarithmic cofine of half A C D; therefore, half A C D is = 48° 42′ 11″, and its difference from ½ B C D will give ½ A C B = 4° 45′ 15″.

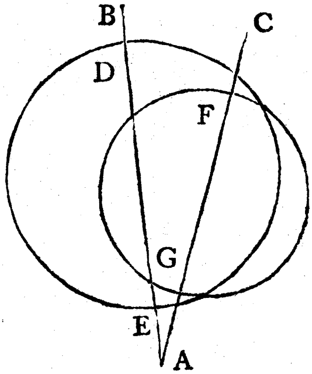
Sine of ½ A C B	8.918444
	8.918444
Sine of A C —	9.767999
Sine of B C —	9.991524
0.0039483 — log. —	7.596411

0.0039483
2

0.0078966
0.7333345 — natural cofine of B C — A C, that is, of 42° 50′.
0.7254379 — natural cofine of A B, or fine of the latitude.
Therefore, the latitude is 46° 30′ 19″.

But farther, as in the cafes, where the above approximation takes place, it is an advantage, that the affumed latitude fhould be taken as near the true one,

one, as can be, the dead reckoning of the ſhip need not be intirely relied on for that purpoſe; for the inſtrumental conſtruction propoſed by Mr. Collins (*d*) will very readily give the true latitude, as nearly as the inſtruments uſed can expreſs: and his method may be deſcribed thus.



Two ſtraight lines AB, AC being drawn in an angle correſponding with the diſtance in time between the obſervations, and in one of the lines, as A B, the lengths A D, A E being taken from any ſcale of tangents, one equal to the tangent of half the ſum, and the other equal to the tangent of half the difference of the diſtance of the ſun from the pole, and from the zenith of the firſt obſervation, and alſo the points F, G, taken in the other line in the ſame manner related to the other obſervation; then circles being deſcribed on the two diameters D E, F G, the diſtance of A from one of the interſections of theſe circles will be the tangent of half the complement of the latitude.

Moreover, as in the treatiſe, which has given occaſion to this diſcourſe, it is propoſed ſometimes to take into conſideration the ſhip's motion during the time, between the obſervations, but imperfectly, regard being had to the change made by the ſhip in longitude only; but the change in latitude alſo may be taken into conſideration in the foregoing con-

(*d*) In the place above cited.

ſtruction,

ſtruction, by making the angle BAC equal to the ſum or difference of the interval of time, and the change in the ſhip's longitude, according as that change is made towards the weſt, or towards the eaſt, and then drawing a line from A, not to the interſection of the circles; but ſo, that the portions of that line, terminated by each of the circles, may be the tangents of arches, whoſe difference is half the change in the latitude; which, if made from the pole, requires the portion of the line terminated at the circle, whoſe diameter is DE, to be ſhorter than the other; and the contrary is required, when the motion of the ſhip is towards the pole.

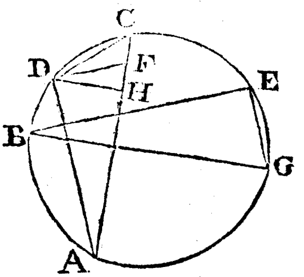
For determining the latitude by calculation, if the diſtance of the ſun in the firſt obſervation from the zenith of the ſecond could be found, this caſe would be reduced to the firſt, wherein the ſhip is conſidered, as ſtationary. And for this purpoſe, it has been propoſed to make an additional obſervation, by an azimuth compaſs, of the angle, the ſhip's courſe makes with the azimuth of the ſun, when the firſt altitude is taken; and, perhaps, the ſame angle may be found with ſufficient exactneſs from the latitude in the firſt obſervation aſſumed, whether from the dead reckoning, or from the foregoing conſtruction.

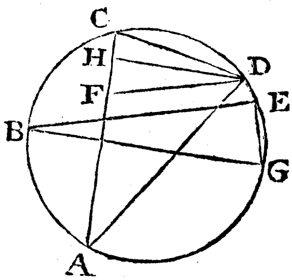
Why I have taken no notice of the calculations exhibited by the author of the piece here animadverted on, as proofs of his method, will readily appear to thoſe, who ſhall caſt their eye upon them.

SCHOLIUM.

The axiom in trigonometry, on which the calculations here diſcourſed of have been ſhewn immediately

diately to depend, was introduced by Regiomontanus, and is ſtill retained, as the foundation of the preſent methods of computing logarithmically an angle from the three ſides of a ſpherical triangle given, though they may be demonſtrated more directly by the following lemma.

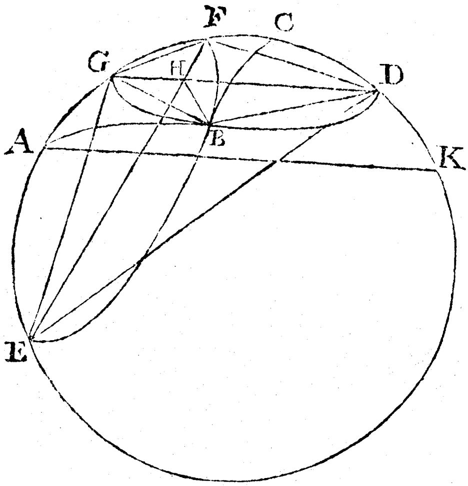




In the circle A B C, the chord A C being drawn, and the arch A B C biſected in B, if D be taken in the circle at pleaſure, and the lines B E, D F drawn alſo at pleaſure parallel to each other, one terminated by the circle in E, and the other by the chord A C in F; then A D, D C being drawn, $BE \times DF$ is $= AD \times DC$.

Draw D H perpendicular to A C, and alſo the diameter B G, joining E G. Then the angles F D H, E B G are equal, and the angles D H F, B E G right. Therefore B E is to B G as D H to D F, and $BE \times DF = BG \times DH = AD \times DC$.

 Now,



Now, in the spherical triangle A B C, through B, and to the poles A, C, let the semicircles E B F, G B D be described, and the arch A C completed to a circle. Then the semicircles EBF, G B D will be both perpendicular to the plane of this circle, cut it in their diameters E F, G D, and their common intersection B H be also perpendicular to it, and consequently perpendicular to G D, the diameter of the semicircle G B D, whereby G B, D B being joined, G D × G H is equal to G Bq, and G D × D H equal to B Dq. Again, A K being drawn parallel to G D, the arch A C K is double A C, G C D double C B, E A D the sum of all the three sides of the triangle, E G = E D − DG (2 CB), GF = AF (AE) + CG (CD) − AC = E D — 2 A C, and F D = E D — E F (2 A B).

Then E G, G F, F D, E D being joined; in the first place, AK is to DG as AK × GH to DG × GH, also AK to DG as AK × DG to DGq, but AK × GH = EG × GF by the preceding lemma, and DG × GH = GBq; whereby AK × DG is to DGq as EG × GF to GBq. Hence AK being twice the sine of the arch AC, DG twice the sine of the arch GC, equal to BC, EG twice the sine of half the arch EAG, GF twice

twice the ſine of half the arch GF, and GB twice the ſine of half the angle ACB to the radius of the circle GBD, which is half GD; ſin. AC × ſin. BC is to rad.^{q} as ſin. $\overline{\frac{1}{2}\,ED - BC}$ × ſin. $\overline{\frac{1}{2}\,ED - AC}$ to $\overline{\text{ſin. }\frac{1}{2}\,ACB}|^q$. And this is Napeir's firſt method of finding an angle from the three ſides given (*e*), as it is uſually delivered.

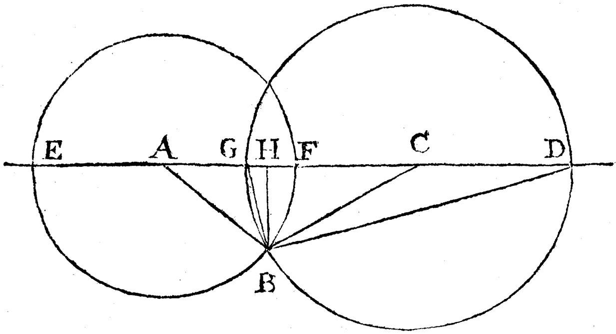
Again, AK is to DG, or AK × DG to DG^q, as AK × DH (ED × DF) to DG × DH, or DB^q. Therefore, DB being twice the ſine of half the angle BCD, or twice the coſine of half the angle ACB to the radius of the circle GBD, as ſin. AC × ſin. CB to rad.^{q} ſo is $\frac{1}{2}$ED × ſin. $\overline{\frac{1}{2}\,ED - AB}$ to $\overline{\text{coſ. }\frac{1}{2}\,ACB}|^q$. And this is Napeir's ſecond method (*f*).

And laſtly, the coſine of an arch or angle being to its ſine, as radius to the tangent, ſin. $\frac{1}{2}$ED × ſin. $\overline{\frac{1}{2}\,ED - AB}$ will be to ſin. $\overline{\frac{1}{2}\,ED - BC}$ × ſin. $\overline{\frac{1}{2}\,ED - AC}$ as rad.^{q} to $\overline{\text{tang. }\frac{1}{2}\,ACB}|^q$, which is a third method added by Gellibrand (*g*).

Moreover,

In plain triangles, from the three ſides given may an angle be found by a proceſs ſimilar to each of theſe, as follows:

(*e*) Mirific. Canon. Log. deſcrip. lib. ii. c. 6. § 8.
(*f*) Ibid. § 9.
(*g*) Trigon. Britan. p. 75.



In the plain triangle ABC, if on the centers A and C be defcribed circles through B, cutting AC in E, F, G, and D, the perpendicular BH being let fall on AC, $EH \times HF$ is equal to $GH \times HD$. Therefore $HF : GH :: HD : HE$, and by compofition $GF : GH :: ED : HE$, and $:: ED - GF : EG$; alfo $HF : GF :: HD : ED$, and $:: DF : ED - GF$.

Moreover ED is $= AB + AC + CB$, $EG = ED - 2CB$, $FD = ED - 2AB$, $ED - GF = EG + FD = 2ED - 2CB - 2AB = 2AC$. Hence GF is to GH as 2AC to EG, whence $GF \times EG$ is $= 2AC \times GH$; and DH is to ED as DF to 2AC, that is, $2AC \times DH$ will be $= ED \times DF$.

Thus $2AC : DG$ ($:: 2AC \times DG : DG^{q}$) $:: 2AC \times GH$ ($EG \times GF$) $: DGH$ (GB^{q}) and $AC \times CB : \text{rad.}^{q} :: \overline{\tfrac{1}{2}ED - CB} \times \overline{\tfrac{1}{2}ED - AC} : \overline{\text{sin.}\ \tfrac{1}{2}ACB}|^{q}$.

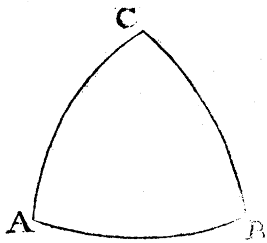
Likewife $2AC : DG$ ($:: 2AC \times DG : DG^{q}$) $:: 2AC \times DH$ ($ED \times DF$) $: GDH$ (DB^{q}); therefore $AC \times CB : \text{rad.}^{q} :: \tfrac{1}{2}ED \times \overline{\tfrac{1}{2}ED - AB} : \overline{\text{cof.}\ \tfrac{1}{2}ACB}|^{q}$.

And in the laft place $\tfrac{1}{2}ED \times \overline{\tfrac{1}{2}ED - AB} : \overline{\tfrac{1}{2}ED - CB} \times \overline{\tfrac{1}{2}ED - AC} :: \text{rad.}^{q} : \overline{\text{tang.}\ \tfrac{1}{2}ACB}|^{q}$.

This laſt propoſition was firſt delivered by Gellibrand, and the other two not long after by Oughtred, in their reſpective treatiſes of Trigonometry.

But it may farther here be noted, that from two ſides of a plain triangle, as AC, CB, with the angle between them ACB, the third ſide AB may be found either by the firſt or ſecond of the foregoing proportions; for by the firſt may be found $EG \times GF$, that is, $AF^q - AG^q$, and by the ſecond $ED \times DF$, or $AD^q - AF^q$; whence by a table of ſquares AF ($= AB$) may be readily found, for AG is equal to $AC - BC$, and $AD = AC + CB$.

In like manner in the ſpherical triangle from the ſides AC, CB, and the angle ACB, may the third ſide be expeditiouſly found with the aid of a table of natural ſines, by the axiom referred to in the beginning of this diſcourſe, as modelled for this purpoſe by Napeir, in the propoſition, which has been already referred to (*h*), whereby the axiom is reduced to this analogy,



$$\text{Rad.}^q : \text{ſin. } AC \times \text{ſin. } CB :: \overline{\text{ſin. } \tfrac{1}{2} ACB}|^q : \text{rad.} \times \frac{\text{verſ. ſ. } AB - \text{verſ. ſ. } \overline{AC \backsim CB}}{2} \ (i).$$

May

(*h*) Mirif. Canon. Conſtr. p. 59.

(*i*) An example of this method has been given above in the triangle ABC in p. 920. And in default of a table of natural ſines, the baſe AB might have been found thus. The ſum of twice the logarithmic ſine of $\frac{1}{2}$ACB, added to the logarithmic ſines of AC and

MAY it here be farther added, that thus all the cases of oblique spherical triangles may be solved without dividing them into rectangular? For when two sides and the angle included, or when two angles and the side between them are given, the two other angles or sides may be found by the proportions of Napeir and Briggs, whereby the other two angles or sides are found together (*k*); the proportions, when two sides with the including angle are given, being these,

$$\text{Cos.}\ \frac{AC + CB}{2} : \text{cos.}\ \frac{AC \backsim CB}{2} :: \text{cotang.}\ \tfrac{1}{2} ACB : \text{tang.}\ \frac{CAB + CBA}{2}.$$

And

$$\text{Sin.}\ \frac{AC + CB}{2} : \text{sin.}\ \frac{AC \backsim CB}{2} :: \text{cotang.}\ \tfrac{1}{2} ACB : \text{tang.}\ \frac{CAB \backsim CBA}{2}.$$

The second of these has been demonstrated with great conciseness by Dr. Halley, from the principles of the stereographic projection of the sphere (*l*); and the first is derived by him from the second, but not

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and BC amounted to 37.596411. The logarithmic sine of $\frac{AC \backsim BC}{2}$ (9.562468) deducted from half this sum (18.798211) leaves 9.235737. This number sought among the logarithmic tangents exhibits for its correspondent sine 9.229400, which being deducted from the foresaid half sum (18.79211) leaves 9.568805, which is the logarithmic sine of 21° 44′ 50½″, equal to half AB.

(*k*) Vide Neper. Mirif. Canon. Constr. pag. penult.

(*l*) See Jones's Synopsis, p. 281.

altogether

altogether with the ſame ſucceſs, by his not obſerving, that if the baſe and one of the ſides be completed to ſemicircles, the ſecond of the two proportions in the ſupplemental triangle thence formed leads directly to the firſt in the original triangle.

And to conclude, if two ſides, and the angle oppoſite to one of them, or two angles with the ſide oppoſite to one, were given; when the other oppoſite part is found from the proportion between the ſines of parts oppoſite, the remaining angle, or ſide, may be found by either of the two proportions foregoing.

LXXXII. *An Account of the Plants* Haleſia *and* Gardenia: *In a Letter from* John Ellis, *Eſq*; *F. R. S.* *to* Philip Carteret Webb, *Eſq*; *F. R. S.*

Dear Sir,

Read Nov. 20, 1760.

YOU muſt have obſerved, that as the ſpirit of planting has increaſed in this kingdom, the ſtudy of botany has become more faſhionable; the works of the celebrated Linnæus, heretofore looked on as capricious and ſtrange, are now in the hands of every man, who wiſhes to ſtudy the order of nature.

The great variety of plants, which you have introduced into your garden from North America, as well as from many other parts of the world, muſt give you double pleaſure, when you view them ranged in proper order, and judiciouſly named.

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